How to Detect the Chandler and the Annual Wobble of the Earth with a Large Ring Laser Gyroscope

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We demonstrate a 16 m^2 helium-neon ring laser gyroscope with sufficient sensitivity and stability to directly detect the Chandler wobble of the rotating Earth. The successful detection of both the Chandler and the annual wobble is verified by comparing the time series of the ring laser measurements against the ‘‘C04 series’’ of Earth rotation data from the International Earth Rotation and Reference System Service.

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The instantaneous Earth rotation rate, usually expressed as variation of the length of day (ΔLOD) and the orientation of the rotational axis of the Earth in space are the quantities which link the terrestrial (ITRF) and the celestial (ICRF) reference frames. Currently a set of very distant quasars, forming an external set of stable reference markers, provide the only way of determining the rotational velocity and the variations of the orientation of the rotational axis of the Earth with sufficient accuracy for precise point positioning and navigation on the Earth's surface. Typical values of less than 10 μs for the measurement of LOD and 0.5 nrad (0.1 mas) for the pole position are routinely achieved by a network of radio telescopes and satellite observations [1,2], by the services of the International Association of Geodesy (IAG). The operation of such a network requires extremely expensive equipment and a massive maintenance effort. Huge amounts of data are recorded in each measurement session, which require physical transport over large distances for the correlation process in the analysis centers. Because of issues such as data latency and the fact that there is no continuous measurement coverage, it is highly desirable to investigate complementary methods for the precise estimation of Earth rotation. Furthermore, the development of independent measurement techniques will allow for the identification of intratechnique biases if they exist.

Ring laser gyroscopes capable of unlocking on the bias provided by the Earth's rotation rate alone were first demonstrated nearly two decades ago [3]. Over the years the lateral dimensions of these devices have increased tremendously. The initial device built at the Cashmere caverns in Christchurch, New Zealand had beam paths which enclosed an area of 0.748 m^2 [3], while the largest ring laser constructed (the UG-2 ring laser) enclosed a massive 834 m^2 [4]. The ring laser equation (1) relates the beat frequency δf of the two counterpropagating continuous wave (cw) laser beams inside the ring cavity to the rate of rotation ($\mathbf{\Omega}$) imposed on the projection on the normal vector ($\mathbf{n}$) of the ring laser structure.

$$\delta f = \frac{4A}{\lambda P} \mathbf{n} \cdot \mathbf{\Omega}. \tag{1}$$

It is clear that increasing the perimeter (P) and hence the area (A) enclosed by the laser beams leads to an increased sensor resolution as well as a reduction of the influence of backscatter induced coupling of the two counterpropagating beams due to imperfections on the surfaces of the cavity mirrors. However this competes with deleterious effects such as additional losses related to the increasing laser beam waists at the cavity mirrors as well as the geometric instabilities arising from the inherently heterolithic cavity design for very large lasers. Ultimately, both resolution and stability become compromised. We show here that a 16 m^2 ring laser with an entirely monolithic zerodur design (the Gross Ring facility in Wettzell, Bavaria) can outperform massive rings in terms of exploitation of the available cavity Q through its considerably enhanced stability. This point is illustrated via observation of the annual wobble and the Chandler wobble of the rotating Earth [5]. The Chandler wobble results from a free mode of oscillation of the Earth [6], which causes polar motion at a very low frequency of approximately 1/435 days corresponding to 26.6 nHz. The annual wobble is caused by small variations of the gravitationally induced torque due to the slightly excentric Earth orbit with a frequency of 1/365 days or 31.7 nHz, respectively.

The Gross Ring (G ring) is situated in Southeastern Germany at the Geodetic Observatory Wettzell (latitude N 49° 08’, longitude E 12° 52’) [7]. The G ring is housed in a purpose built underground laboratory. The monument supports a polished granite table upon which the laser rests with protection against external influences. The laser itself is made of zerodur. Four bars are rigidly attached to a base plate forming the edges of a square with 4 m side length yielding a free spectral range of 18.75 MHz. The coefficient of thermal expansion is 1.4×10^{-8} K^{-1} for the base plate and -1.7×10^{-8} K^{-1} for the bars, resulting in a total coefficient of less than 1×10^{-8} K^{-1}. The four mirrors and their mirror holders are attached at the face sides of the bars by molecular adhesion. This technique ensures a stable vacuum seal. The mirrors are of extreme quality, having losses of only a few parts per million. The laser medium, a helium-neon gas mixture with a pressure of a few hPa, is excited in a Pyrex gain tube of 5 mm diameter by radio-frequency excitation to maintain lasing at 632.8 nm. The high frequency excitation scheme eliminates biases from nonreciprocal effects on the laser beams by avoiding gas flow.

Diurnal polar motion and solid Earth tides, although much smaller in amplitude than the Chandler wobble, have been accessible with large ring laser gyroscopes for several years [8,9]. By contrast, neither the Chandler wobble with a period of ≈ 435 days nor the annual wobble with a period of 365.25 days, have been separable from the remaining sensor drift due to the extreme demands on the long term stability of the sensor hardware. Specifically, this arises because of the continuous variations in atmospheric pressure with a maximum of $\Delta P \approx \pm 20$ hPa and the seasonal periodic change of temperature with a peak to peak value of nearly $0.5°$ C corresponding to $\Delta T \leq 4$ mK/day, which are sources of considerable instability. In addition to these environmental influences, other instrumental effects like, photon noise, laser gas aging and limitations to the monumentation stability of the entire apparatus can cause variation of the effective scale factor of the sensor (including backscatter effects) and are known as the major sources of instabilities of small navigational gyros, which have reached values as low as 200 μ deg/$\sqrt{\text{h}}$ for the *Honeywell GG 1389* for example. These latter effects have been mostly excluded by the rigid sensor design [7] and by including a getter pump in the resonator to remove hydrogen. The G ring now routinely reaches values for the random walk error of less than 1.3 nano-deg/$\sqrt{\text{h}}$ as long as the backscatter coupling remains constant. Under these circumstances, atmospheric pressure variations are the most important sources of instability. We have observed shifts in the optical frequency of the ring laser resonator of as much as 330 kHz per hPa. With a free spectral range $c/f = 18.75$ MHz and the scale factor from Eq. (1) defined as $S = \frac{4A}{\lambda P}$ this corresponds to a variation of $S \approx 7 \times 10^{-10}$, which cannot be neglected. Figure 1 shows the environmental effects with a peak around 1 day ($\approx 10^5$ s) in period in the 2006 Allan deviation (ADEV) estimate of the ring laser performance. To overcome this, we have encapsulated the entire ring laser structure with a pressure stabilizing vessel. Using a frequency comparison between one of the beams of the G ring and the output of a Winters model 200 iodine stabilized reference laser, perimeter changes of the G ring are determined. Each frequency assessment and pressure adjustment took about 5 min to be evaluated. This integration is necessary because of the low light level extracted from the ring laser (10 nW) and the feedback jitter of the actively stabilized reference laser. A feedback system adjusts the pressure inside the vessel such that a constant cavity perimeter is maintained. Apart from isolating the cavity from atmospheric pressure variations, we also compensate for small thermal expansions from the seasonal temperature cycle of the entire sensor structure. In addition to this, we also stabilize the relative power of the two counterpropagating intracavity beams via a feedback mechanism. This minimizes any possibility for nonreciprocal effects within the gain medium to introduce a systematic variation to the measured Sagnac frequency.

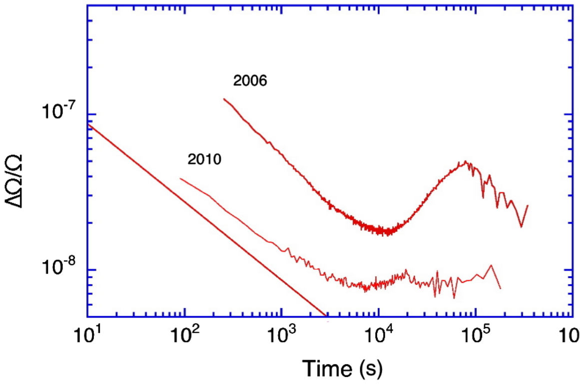

FIG. 1 (color online). ADEV of the sensor resolution of the G ring laser in 2006 and with the new set of mirrors plus pressure stabilization in 2010. The straight line to the left of the plot indicates the calculated quantum limit of the G ring.

Statistically the system is able to maintain the preset optical frequency to within a standard deviation of 60 kHz for at least three months. Further enhancements to the laser performance were obtained through the use of the latest generation supermirrors supplied by Research Electro-Optics Ltd. These mirrors both utilize a fused silica substrate (for which a superior polish can be achieved) and have improvements to the coating technology to allow greater homogeneity over the clear aperture. Figure 1 shows an ADEV of the current 2010 performance level of the G ring laser against the earlier results obtained in 2006.

There are several conclusions to draw from Fig. 1. In 2006, when there was no pressure vessel in use, atmospheric pressure variations were able to cause variations in the measurement signal, which are more than 1 order of magnitude larger than geometrical scale factor variations through pressure induced compression of the ring laser body alone. As a result we observe a hump in the ADEV for integration times which are comparable to the time span on which weather patterns typically change in Central Europe. This is thought to be caused by small variations in the phase of the backscatter coupling of each beam, which are stimulated by the minute atmospheric pressure induced change in length of each arm at the level of less than 1 nm out of a total length of 4 m. If all changes in distance between the four mirrors were entirely symmetric, no backscatter variation would result, because dimensional changes and the corresponding shift in optical frequency would compensate for each other. From that point of view, we are only seeing the differential component of the changes in the length of the four arms of the square cavity. With the pressure stabilizing vessel in operation, most of this backscatter perturbation disappears as is apparent from the 2010 data. Now consider the ’irreducible’ noise limit set by quantum mechanical phase instabilities. The G ring has a ringdown time of $\tau = 1.2$ ms, the quality factor of the cavity $Q = \omega\tau$ is as high as 3.5×10^{12}. Then, according to [10], the quantum noise limit on the resolution of the gyroscope can be written as

$$\Delta\Omega = \frac{cP}{4AQ}\sqrt{\frac{hf}{p_o t}}, \tag{2}$$

where P is the perimeter, A the area, f the optical frequency, h the Planck constant, p_o the optical power loss and t the time. Thus, in the complete absence of backscatter, the G ring would reach a relative sensitivity of $\Delta\Omega/\Omega_E = 10^{-9}$ within 3 h, where Ω_E is the rate of rotation of the Earth at a latitude of 49° N. However, due to small variations in residual backscatter induced frequency shifts at low frequencies well below 1/day, the resolution of the G ring levels out in a flicker floor at a value just below 10^{-8} of $\Delta\Omega/\Omega_E$. It is this extreme sensitivity coupled with high stability which can be exploited to measure small ac signals of geophysical origin and can be expanded to include those which occur on longer time scales of hundreds of days, such as the Chandler wobble.

A ring laser gyroscope primarily measures rotation. Theoretically G achieves a resolution of $\Delta\Omega = 1.2 \times 10^{-11}\,\mathrm{rad/s}/\sqrt{\mathrm{Hz}}$ as calculated above [Eq. (2)]. Additional contributions to the measurement signal arise from variations of the projection of the instantaneous Earth rotation axis onto the normal of the laser plane, which is expressed as the inner product between the normal vector $\mathbf{n}$ and the rotation vector $\boldsymbol{\Omega}_E$ in the ring laser equation [Eq. (1)]. This angular variation can be divided into a local component describing the instrumental orientation with respect to the Earth’s body and variations in latitude of the Earth rotation axis itself, that is the polar motion. In the local regime only north-south tilts affect the horizontally mounted G ring laser. In the case of polar motion the instrument is sensitive to the component along its meridian circle only. As polar motion is usually given as an angular offset along the Greenwich (x_p) and 90° west meridian (y_p), the effective polar motion Θ_{pm} at the instrument’s longitude α_0 is

$$\Theta_{\mathrm{pm}} = \cos(\alpha_0)x_p - \sin(\alpha_0)y_p. \tag{3}$$

The minus sign is a consequence of historical conventions, which defined y to point towards 90° west. The ring laser senses both, the local north-south tilt Θ_{loc} and polar motion as an apparent latitude variation $\delta\phi$ such that the inner product in Eq. (1) can be written as

$$\mathbf{n}\cdot\boldsymbol{\Omega} = |\Omega|\sin(\phi + \Delta\phi), \tag{4}$$

with

$$\Delta\phi = \Theta_{\mathrm{loc}} + \Theta_{\mathrm{pm}}. \tag{5}$$

Local tilts are permanently monitored to a high accuracy (better than 1 nrad) and applied as a correction term to the G ring laser time series, so that the lower curve in Fig. 3 does not contain any local tilt effect. Polar motion however can only be extracted when additional signal information is used. For the case of diurnal polar motion a precise model is available [11]. For the combined effect of the Chandler and the annual wobble however a time series of measurements from geodetic space techniques in the form of the EOP C04 series of the IERS [12] is available.

A typical data set for the G ring laser with the stabilization of the cavity length in operation contains a measurement noise level of nearly 2 prad/s while the overall drift roughly stays within ±5 prad/s. To obtain such a data set, we have taken the raw measurements and integrated them over 0.5 h for each data point. Then we used locally measured tilt data to correct the measurements for locally observed tilt effects [9]. We then computed the effects of polar motion from the appropriate model and subtracted them from the ring laser measurements [8].

Figure 2 shows the Chandler wobble as it was observed by VLBI between July 2009 and December 2010. The data is taken from the C04 series of the IERS. The ring laser observations were taken in a time period where the main

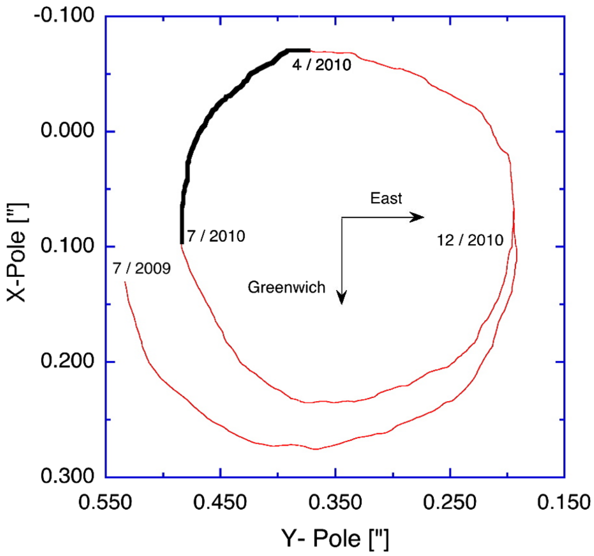

FIG. 2 (color online). Polar motion observed with VLBI as reported by the IERS. The data set covers half of the year 2009 and the entire 2010. A value of 0.1 s of arc at the pole roughly corresponds to 2 m. The data section between April and July 2010 (marked with a thick line) was employed as the reference data for the ring laser measurements.

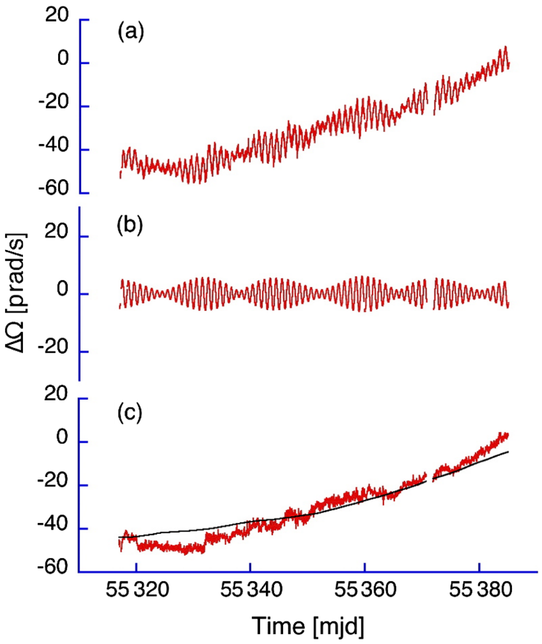

FIG. 3 (color online). G ring measurements of the Chandler wobble. (a) The ring laser observations over the measurement period with the constant Earth rotation rate removed and with local tilt corrections applied. (b) The calculated contribution from the diurnal polar motion. (c) The combined effect of the Chandler and annual signal calculated from the C04 series data from VLBI measurements (solid line) and the G ring data.

contribution was along the X-pole direction, because this creates the largest signal amplitude in the ring laser data. Over the time of observation, backscatter effects within the laser remained essentially constant.

Figure 3(a) shows the experimental data from the ring laser over the measurement period with the constant Earth rotation rate removed and with local tilt corrections applied. The sinusoidal signal (b) is the calculated contribution from the diurnal polar motion, which is a well known signal as discussed above and matches the observation very well. The solid line (c) is obtained using the C04 series data to compute the change of projection $\delta\mathbf{n}$ of the instantaneous Earth rotation axis from a constant initial value relative to the ring laser G. This is then substituted into the ring laser equation as a function of time. As can be seen, this yields excellent agreement with the slowly increasing rotation rate experienced by the G ring laser.

It is notable that aside from some aperiodic residual signal variations, the data appears to contain a linear drift of 0.164 (prad/s)/day over almost 60 days, which is the highest sensor stability the G ring has ever achieved. These drift effects arise from slow variations in the backscatter phase due to the small seasonal drift in ambient temperature and is currently being addressed.

The sensitivity and stability of large ring laser gyroscopes has improved so dramatically that we are now able to directly measure the combined effect of the Chandler and the annual wobble of the freely rotating Earth. The ring laser data is in excellent agreement with the independent measurements by VLBI. To achieve the required stability, the optical frequency was stabilized to within 60 kHz for a period of three months.

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