Improved Kennedy-Thorndike Experiment to Test Special Relativity

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We have carried out a modern version of the Kennedy-Thorndike experiment by searching for sidereal variations between the frequency of a laser locked to an I_2 reference line and a laser locked to the resonance frequency of a highly stable cavity. No variations were found at the level of 2×10^{-13}. This represents a 300-fold improvement over the original Kennedy-Thorndike experiment and allows the Lorentz transformations to be deduced entirely from experiment at an accuracy level of 70 ppm.

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The possibility of using lasers to improve the accuracy of the classical experiments[1-3] of special relativity (SR) was originally suggested by Javan and Townes and in fact they were the first to make a more precise Michelson-Morley experiment.[1,4] The full potential of modern laser-frequency metrology for length measurements was, however, not exploited until the more recent precision Michelson-Morley (MM) experiment of Brillet and Hall[5] which achieved a fractional frequency uncertainty of $\pm2.5\times10^{-15}$ in showing the isotropy of space. They noted also the technical difficulties which would have to be overcome to achieve similar large improvements in a laser version of the Kennedy-Thorndike (KT) experiment,[2] which utilized an interferometer of unequal arm lengths to compare the transformations of time and length in a moving frame. We have performed the physically equivalent measurement by searching for a sidereal 24-h variation in the frequency of a stabilized laser compared with the frequency of a laser locked to a stable cavity. Our measurements yield a sensitivity of $\sim2\times10^{-13}$ for a sidereal term, corresponding to a $\approx$300-fold-higher accuracy than the original KT result.

Following Robertson,[6] Mansouri and Sexl[7-9] have developed a useful framework for explaining what an experiment measures and how it relates to other experiments. They consider two coordinate systems in relative motion, and write the most general transformation between Σ (the preferred frame) and S (the moving frame):

$$t=a(v)T+ex,\quad x=b(v)(X-vT)\,,$$
$$y=d(v)Y,\quad z=d(v)Z\,. \qquad (1)$$

In these equations e is fixed by synchronization procedures for clocks while the kinematical parameters $a(v)$, $b(v)$, and $d(v)$ might be determined by theory but, more importantly, can be determined by experiment. Because of isotropy in Σ, these transformation parameters are even functions, dependent only on $(v/c)^2$, i.e., $a=1+\alpha(v/c)^2+\cdots$, $b=1+\beta(v/c)^2+\cdots$, and $d=1+\delta(v/c)^2+\cdots$. Apart from synchronization, transformation between the preferred frame Σ and a moving frame S is thus completely specified by the parameters α,β,δ. In general, in the frame S the velocity of a light ray depends on the direction of propagation. For a light ray propagating at an angle θ with respect to the x axis (parallel to $\mathbf{v}$), the velocity follows[7,8,10] from Eq. (1):

$$c(\theta)=[1+(\tfrac{1}{2}-\beta+\delta)(v/c)^2\sin^2\theta + (\beta-\alpha-1)(v/c)^2]c\,. \qquad (2)$$

Special relativity states unambiguously $\alpha=-\frac{1}{2}$, $\beta=\frac{1}{2}$, $\delta=0$, corresponding to $c(\theta)=c$ for all frames.

Present knowledge of the parameters α,β,δ (which also quantifies the degree of agreement of Einstein's SR and observation) comes principally from the second-order MM and KT optical experiments and from Mössbauer rotor and optical experiments which determine the time dilation parameter α. (For a more detailed discussion of the many excellent experiments, see Refs. 7-9 and 11.) The most accurate time-dilation experiments[12-14] imply $\alpha=-\frac{1}{2}\pm1\times10^{-7}$ and the most accurate MM experiment[5] determines $\beta-\delta=\frac{1}{2}\pm5\times10^{-9}$. The original KT experiment[2] leads to $\alpha-\beta=-1\pm2\times10^{-2}$ which thus introduces the single greatest uncertainty in the transformation equations. For these reasons, the importance of improving the KT experiment has been stressed repeatedly.[9,15,16]

Modern astrophysical measurements[17] of anisotropy in the microwave background (MWB) seem to define a universal standard of rest which reasonably could be taken to be the preferred frame Σ. In the following we will assume that the relevant velocity v is given by the observed motion of Earth with respect to the MWB frame. The signature of the effect we search for is its dependence on the sidereal modulation of v due to the Earth's rotation, yielding a 24-h sidereal term.

The KT experiment can be viewed as a differential comparison between a standard of time defined by a mercury lamp and a standard of length in the form of an unequal-arm Michelson interferometer. Our physically equivalent experiment utilizes instead two He-Ne lasers, one locked to a molecular absorption line in I_2 [$R(127)$ 11-5], while the other is locked to a very stable Fabry-

Pérot reference cavity. Their frequencies are compared by optical heterodyne detection. From Eq. (2), we determine[10] the cavity-locked-laser frequency ν_1^S in the moving frame S:

$$\nu_1^S/\nu_c = 1 + (\beta - \alpha - 1)(v/c)^2 + (\delta - \beta + \tfrac{1}{2})(v/c)^2 \sin^2\theta \,, \quad (3)$$

where $\nu_c = pc/2L$, p is an integer, and L is the length of the Fabry-Pérot interferometer as measured in S. Let ν_2^S denote the I_2-stabilized-laser frequency. Then the heterodyne beat of the two optical frequencies is

$$\nu_{\rm beat}^S/\nu_c = 1 + (\beta - \alpha - 1)(v/c)^2 + (\delta - \beta + \tfrac{1}{2})(v/c)^2 \sin^2\theta - \nu_2^S/\nu_c \,. \quad (4)$$

The precise MM result of Brillet and Hall[5] implies $\delta - \beta + \frac{1}{2} = 0 \pm 5\times 10^{-9}$ and so to this accuracy

$$\nu_{\rm beat}^S/\nu_c = 1 + (\beta - \alpha - 1)(v/c)^2 - \nu_2^S/\nu_c \,. \quad (5)$$

Working in the preferred frame Σ, one can show that

$$\nu_{\rm beat}^\Sigma/\nu_c = [1 + \alpha(v/c)^2]\,\nu_{\rm beat}^S/\nu_c \,,$$

a result to be expected due to the time dilation effect.

We next determine the velocity v of our laboratory with respect to the MWB frame. Working in the Earth equatorial frame we find the main sidereal components

$$(v/c)^2 = (u/c)^2 + 2(u/c)(\Omega R_\oplus/c) \times \cos\phi_L \cos\delta_\mu \sin[\Omega(t - t_s) + \Phi] \,. \quad (6)$$

In this equation $u = 377 \pm 14$ km/s is the velocity of Earth with respect to the MWB frame,[17,18] $\Omega = (2\pi/P_\oplus)$ with $P_\oplus$ being the sidereal period and $R_\oplus$ the Earth radius, $\phi_L = 40°$ is the latitude of Boulder, $\delta_\mu = -6.4° \pm 1°$ is the observed declination of the MWB velocity vector,[17,18] and Φ is the phase at the start of the analysis epoch. From Eqs. (5) and (6) we see that determination of the factor $\beta - \alpha - 1$ depends on our ability to measure the 24-h sidereal variation of the beat frequency.

The principle of our experiment is discussed with reference to Fig. 1. A He-Ne laser ($\lambda = 6328$ Å) is locked[19] to a highly stable, isolated Fabry-Pérot interferometer, thereby satisfying optical standing-wave boundary conditions. The servo system then transforms length variations of the cavity (of accidental or cosmic origin) into laser-frequency variations. These can be sensitively detected by optically heterodyning some of the laser power with an optical frequency reference provided by an I_2-stabilized laser. The beat frequency [$\sim$160 MHz if the I_2-stabilized laser is locked to the d component of the $R(127)$ 11-5 transition] is counted for 40 s and stored with negligible dead time. We usually acquire 4320 beat-frequency readings (=2 days) in memory before storing the data on disk and restarting.

Our fundamental standard of length is the Fabry-Pérot interferometer. It uses Zerodur "gyro"-quality

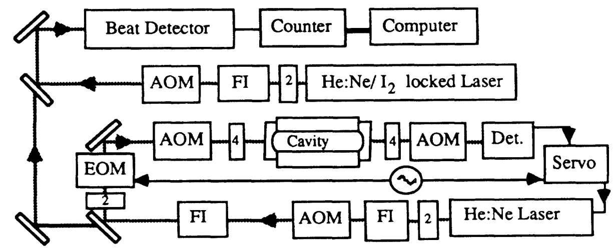

FIG. 1. Laser-based Kennedy-Thorndike experiment. A He-Ne laser is locked to a transmission fringe of a highly stable Fabry-Pérot cavity. The heterodyne beat between it and a second He-Ne laser, stabilized to I_2, is measured in 40 s intervals and stored in the computer memory. After 2 days (4320 points), the beat-frequency and temperature data are stored to disk and the experiment reinitialized. FI means Faraday isolator. The thin wave plates are marked in inverse fractional waves.

mirrors optically contacted to the ends of a Zerodur spacer (length 30 cm, diameter 15 cm). The radii of curvature are $R = 575$ cm and $R = \infty$; transmission is $T = 30$ ppm. The cavity fringe width is 72 kHz FWHM, giving a finesse $F = 6600$, and a resonant transmission of $\sim$2%. The interferometer is suspended by two stainless-steel ribbons (1×0.01 cm^2), one at each end, inside a thick-walled vacuum envelope. A low-drift servo loop with ac-thermistor sensing stabilizes this aluminum-wall temperature to better than 5 μK over a 1-day period. The experiment is located inside the "quiet house" which provides a > 20-dB thermal and acoustic barrier.[20] The inside air temperature is stabilized by Peltier-cooled panels which pump the laser discharge heat out via a slow water flow. This yields a stability better than 1 mK over times of 1 h and better than 10 mK over a 1-day period, in the face of room-temperature variations of $\pm$1 deg.

The "hard-seal" He-Ne laser provides about 1 mW of red light at 6328 Å. The beam passes through an isolation stage formed by a pair of Faraday isolators and one acousto-optic modulator (AOM). An ammonium-dihydrogen-phosphate phase-modulator crystal (EOM) is driven at $\sim$1 kV peak to peak at a modulation frequency of 25 kHz via a resonant step-up transformer. Additional isolation (via frequency shift) of the laser beam reflected by the Fabry Pérot interferometer is provided by a second AOM. The transmitted light is frequency shifted by a third AOM to avoid fringes of the photodetector's scattered light and the output mirror of the cavity. The ac output of the photodetector, after lock-in detection, provides the error signal for the servo system. Based on the 10 μW of fringe signal and a unity gain frequency of 5 kHz, the shot-noise limit of the cavity-locked laser is expected to be $\sim$1 mHz, while the observed[19] frequency noise at short times ($\sim$1 s) is less than 50 mHz using first-harmonic detection.

The I_2-stabilized He-Ne laser which serves as our optical frequency standard achieves a stability of ~500 Hz in 40 s due to the finite signal-to-noise ratio of the ~0.1% I_2 saturation peak in ~100 μW of laser power. The long-term stability is ~100 Hz. In contrast, the cavity length has excellent short-term stability, but long-term changes arise from internal processes (creep) and environmental thermal effects. Because of aging effects (shortening) of the Zerodur spacer, the beat between the two lasers exhibits a long-term uniform drift of 1.65 Hz/s (3×10^{-10}/day or 1.1×10^{-7}/yr) beginning in July 1986, down to 1.06 Hz/s in March 1989. Our observed drift agrees with the aging curve measured by Bayer-Helms, Darnedde, and Exner[21] for Zerodur gauge blocks. The cavity frequency is predictable within 1 Hz for 1000 s and within < 300 Hz for 1 day.

Figure 2 shows a two-day segment of the recorded heterodyne beat frequency. The uniform Zerodur creep, about 185 kHz, has been subtracted. We suppose residual temperature changes working via the I_2 pressure shift may cause part of the small slow variations.

Our first KT data begin on 27 July 1986 and end on 22 September 1986. This includes ~15 days of uninterrupted data. Another block of data begins 31 October 1988, ends 30 April 1989, and includes more than 90 days of uninterrupted data. Analysis proceeds as follows: We first average the 4320 beat-frequency samples (=2 days) in blocks of 9 which reduces the sampling rate to 10/h. We next remove from the 2-day data set the uninteresting linear trend from cavity creep. We note this procedure may remove some power (< 10%) from a hypothetical sinusoidal signal at 1 cycle/day. We then decimate the data set (like Fig. 2) to a final sample rate of 1/h. This leaves us with the remaining frequency residuals due to uncontrolled environmental pertubations, as well as to a possible "aether effect."

We have examined the frequency residuals for a sidereal signal by several methods, with similar results. Here we Fourier analyze the full nearly 3-yr-long record by putting zero values into the data gaps. [The amplitude scale has been corrected (×11.3) for the attenuation due to the zero-fill procedure.] The spectral power is reduced below $\frac{1}{2}$ cycle/day by our removal of the drift term and above ~3 cycles/day due to the low pass filtering by the cavity's 1-day thermal isolation time.

Examination of Fig. 3(a) indicates an enhanced noise level near 1.0 cycle/day. It is easy to imagine a strong driving term at the solar frequency, perhaps phase shifted and broadened by variables such as cloud cover, weekend work schedules, etc. With breaks in the data, the usual windowing procedure is ineffective, so even a bright line input will corrupt adjacent frequency bins. To find the transfer function, we tried adding a strong solar or sidereal signal in the time domain. The half-amplitude Fourier-transform width was ±3 bins. Removing the best solar sine wave drops the sidereal amplitude from 33.1 to 14.9 Hz [see Fig. 3(b)].

To test if the increased noise level was due to leakage from a "real" signal at the sidereal frequency, we removed this best-fitting sine wave in the time domain as before. The spectrum was essentially unchanged from Fig. 3(a). We conclude that the broad noise buildup around 1 cycle/day is due to solar—not sidereal—input.

For this report we work conservatively with the unmodified data of Fig. 3(a). Working near (±4 bins) the sidereal bin, the quadrature amplitudes are found to be normally distributed, with standard deviation $\sigma_n = \langle A_c^2\rangle^{1/2} = \langle A_s^2\rangle^{1/2} = 30.3$ Hz. The sidereal amplitude, 33.1 Hz, gives a normalized measured value $x_m = 33.1/30.3 = 1.09$. It is composed of an 11-Hz component along the MWB axis, and 31 Hz perpendicular to this direction.

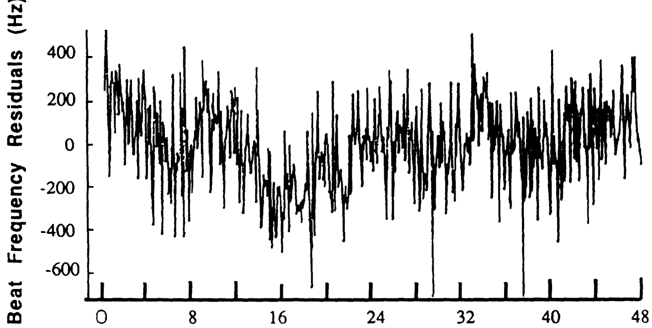

FIG. 2. Heterodyne-beat-frequency residuals. Uniform cavity drift of 1.08 Hz/s is removed. Fast noise is from the I_2 reference. Slow drifts may be partly thermal.

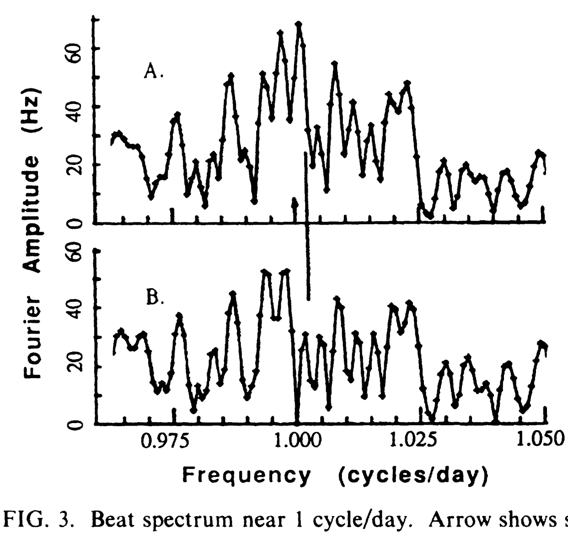

FIG. 3. Beat spectrum near 1 cycle/day. Arrow shows solar frequency, 1 cycle/day, corresponding to bin 1097. Vertical line marks sidereal frequency, 1.00274 cycle/day, corresponding to bin 1100. (a) Fourier transform (FT) of 3 yr of data. (b) Solar term of 49.4-Hz amplitude removed from the time-domain data before FT. Sidereal amplitude drops from 33.1 to 14.9 Hz (see text).

We now wish to analyze our results with the hypothesis that we have a sinusoidal signal of amplitude P in the presence of random noise, a case which has been considered by Rice.[22] The normalized sinusoidal amplitude $a=P/\sigma_n$. From the Rice function, $p(x_m,a)$, we find there is $<10\%$ probability for finding a realized value $x=x_m$ for $a>2.1$. Thus we would conclude that the probability is less than 10% that there was a real signal as large as $x=a$ units of σ_n, i.e., $P<64$ Hz.

However, inspection of Fig. 3(a) shows that one local peak ($A=65$ Hz) falls in bin 1094, just conjugate to the sidereal frequency of interest, bin 1100, i.e., symmetric around the solar bin (1097). We offer the following, less optimistic scenario: Suppose that the modulation processes (weather, cloud cover, etc.) that spread out the solar forcing function produce symmetrical "sidebands" around the solar bin. Then we can view the upper "weather" sideband, also of $A=65$ Hz, as partially canceling a putative sidereal amplitude to give the realized value of 31.7 Hz. A pessimistic estimate would assume ideal out-of-phase cancellation, producing the values $65+31.7=96.7$ Hz for this sidereal amplitude. We regard this scenario as less than 10% probable, and can therefore believe that *our experiment shows, with a probability >90%, that there is no sidereal signal as large as 96.7 Hz (rounded to 100 Hz).*

The present result, setting an upper limit to a possible "aether effect" amplitude of $P<100$ Hz, corresponds to a fractional frequency amplitude $\Delta\nu/\nu<2\times10^{-13}$. From Eqs. (3) and (4) our experimental result can be expressed in the form $2(\beta-\alpha-1)u<50$ m/s. Using the value $u=377$ km/s one obtains $\beta-\alpha-1<6.6\times10^{-5}$. This limit enables us to deduce separately $\beta=\frac{1}{2}\pm7\times10^{-5}$ and $\delta=0\pm7\times10^{-5}$, to be taken with the already known value[12-14] $\alpha=-\frac{1}{2}\pm1\times10^{-7}$.

Another "dissimilar clocks" experiment was performed by Turneaure *et al.*[23] using superconducting-cavity-stabilized oscillators heterodyned with a pair of hydrogen-maser clocks. Their fractional frequency variations were also limited by environmental considerations to levels approximating those reported here, while their analysis concerned testing the equivalence principle by using the time-varying solar gravitational potential.

This report summarizes an improved Kennedy-Thorndike-type experiment based on modern laser metrology. The heterodyne signal between a cavity-locked He-Ne laser and one stabilized on I_2 shows a fractional frequency of $\Delta\nu_{beat}/\nu_c<2\times10^{-13}$ (90% confidence interval). This null result is 300-fold more accurate than the previous best measurement, made by Kennedy and Thorndike in 1932. Following the reasoning of Robertson,[6] the Lorentz transform of SR can now be based on experimental facts at the 70-ppm level. Another 100-fold improvement might be possible in orbit.

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[10]We note misprints in Ref. 8: d^2 should read d^{-2} in Eq. (6.15) and Eq. (6.17). Similarly δ should read $-\delta$ in Eq. (6.18).

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