GALACTIC CHAOS AND THE CIRCULAR VELOCITY AT THE SUN

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ABSTRACT

Our galaxy has a substantial nucleus that exhibits itself as a strong rise in the rotation curve inside of 2 kpc, peaking around 500 pc. Stars with very eccentric orbits that pass through the nuclear region generally cannot be confined to a flattened disk distribution; instead, they spend most of their time in the halo. In a local sample of high-velocity-dispersion disk stars, there should be an apparent deficiency of stars with very low angular momenta, because they are scattered to much higher scale heights. The deficiency or gap will be centered on the circular velocity as reflected in the motion of the LSR. Using a new galactic model, we calibrate the predicted depression in the distribution of tangential velocities, finding a half-width of 40 km s^{-1} with a depth greater than 80%. We present evidence that the expected deficiency of low-angular-momentum stars does exist in the local stars. The strength of the conclusion is limited by the small size of the current sample of appropriate intermediate-population stars: those that are both in a strongly flattened distribution and possess significant numbers of members at low angular momentum. The available data favor a scale and model-free circular velocity at the Sun in the range 225–245 km s^{-1} with a most probable value of 235 km s^{-1}. Larger samples that are forthcoming will allow this simple method to be applied with much greater confidence.

I. INTRODUCTION

The galaxy's circular velocity at the radius of the Sun, V_0, is a fundamental astronomical quantity necessary for the study of motions of both stars and galaxies. There are two usual types of circular velocity determination: direct measurement against some slowly rotating system such as halo stars or local galaxies, and indirect measurements that require a knowledge of some auxiliary quantity. The accuracy of the determined V_0 is then largely dependent on the values of these external quantities. Estimates of V_0 range from 180 to 280 km s^{-1} (e.g., Mihalas and Binney 1981).

We propose a simple, direct method to measure the circular velocity. The main premise is that there is a substantial nuclear bulge in our galaxy. In practice, we also require a knowledge of the motion of the Sun with respect to the LSR. Low-angular-momentum disk stars plunge into the nuclear-bulge region and have chaotic orbits (Martinet 1974). The practical consequence is that such low-angular-momentum stars cannot be confined in a strongly flattened distribution. Therefore, in a local sample of disk stars, an apparent deficiency of low-angular-momentum stars will arise, the missing stars spending most of their time in the halo. The distribution of tangential velocities will have a depression, centered at the velocity of zero angular momentum as reflected in the solar motion. This method for determining V_0 is dependent only upon the existence of low-angular-momentum chaotic orbits. Galactic structure works against the application of this idea, providing relatively few low-angular-momentum stars in a flattened distribution.

The plan of this paper is as follows. A galactic potential model is presented in Sec. II, and used in Sec. III to integrate representative orbits. The expected depth and width of the gap in the tangential velocity distribution are calibrated using this model. In Sec. IV, catalogs of local samples of stars with good space velocities are examined and found to exhibit the gap. The primary source of uncertainty is the small number of appropriate stars. The gap has a modest statistical significance, and is centered at a velocity of 235 km s^{-1} with an uncertainty of 10 km s^{-1}. A discussion of further systematic uncertainties is in Sec. V, along with our conclusions.

II. A GALAXY MODEL

There is no shortage of galaxy models available in the literature. A useful comparative survey of models is included in the work of Caldwell and Ostriker (1981). A recent standard model is presented in Bahcall, Schmidt, and Soneira (1982). These galactic models have the disadvantage that the calculation of orbits is somewhat awkward and time consuming because the models are constructed from density functions that have complex mathematical forms for the associated potential. Numerical evaluation of these functions can be costly, especially for the very eccentric orbits of interest here that require frequent evaluations of the forces to maintain accuracy. Our model is based on simple potentials that sacrifice only a little of the density detail found in the standard models.

A simple, flattened potential function that meets our needs has been used previously by Miyamoto and Nagai (1975) and Clutton-Brock, Innanen, and Papp (1977). The data fitted to the parameters in those models has been supplemented with more recent information that calls for an update. The model is composed of a disk-halo potential supplemented with spherical potential. The simplest disk-halo potential is a generalization of a Plummer potential, i.e., in cylindrical coordinates,

$$\phi(r,z) = -\frac{\mathscr{M}}{([a+(z^2+h^2)^{1/2}]^2+b^2+r^2)^{1/2}}, \quad (1)$$

where $\mathscr{M}$ is the mass, a is the scale length of the disk, h corresponds to the disk scale height, and b is the core radius of the halo component. The potential's shape can be adjusted through the addition of more disk components, i.e., generalizing $\sqrt{(z^2+h^2)}$ to $\Sigma\,\beta_i\,\sqrt{(z_i^2+h_i^2)}$, where the β_i sum to 1. Miyamoto and Nagai (1975) and Clutton-Brock *et al.* (1977) discuss methods for determining the parameters

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from observational data.

We provide an updated galaxy model here, the main difference from the earlier models being the addition of a massive halo component that maintains a flat rotation curve at large radii. This feature is relatively unimportant for the orbits of interest in this paper. The most important constraint on the model is the rotation curve of the galaxy. We use the central rotation velocities summarized in Oort (1977), between 3 kpc and the Sun the smoothed data presented by Burton (1975), and a nearly flat rotation curve beyond the Sun (Knapp *et al.* 1979; Schechter 1986). The model has $V_0 = 235$ km s^{-1} and $R_0 = 8.5$ kpc. There are two additional, completely spherical components. The nucleus has a scale length of 250 pc, and a spherical bulge with a core radius of 3.0 kpc is added to prevent the rotation curve from falling below 200 km s^{-1} between 1.5 and 3 kpc.

The flattening of the central plane in the model is determined from the thickness and mass density of the thin disk in our galaxy. The three components of the disk represent, respectively, the old disk, the young disk, and an additional dark component to make up the rest of the total solar-neighborhood mass density to a total of 0.186 $\mathscr{M}_\odot$ pc^{-3} (Bahcall 1984a,b). The fractional proportions of old disk, dark matter, and young disk are given by β_1, β_2, and β_3, respectively. Fitting these constraints to the model parameters is a strongly nonlinear process, the details of which are not important here. The parameter data for the model are given in Table I and the rotation curve is shown in Fig. 1.

III. ORBIT INTEGRATIONS

The stellar orbits in this galactic potential come in three distinctly different types. High-angular-momentum orbits with relatively low vertical velocities tend to be tubes, the descendants of the circular orbit. Relatively high-angular-momentum orbits with higher vertical velocities are usually boxes. Both of these types of orbits have as conserved quantities their energy, the z component of the angular momentum, and some third integral or quasi-integral. As a crude approximation, the third integral in an axisymmetric flat galaxy can often be taken as the vertical oscillation energy. Consequently, the vertical motions are nearly decoupled from the radial motions, and disk stars are constrained to remain in a strongly flattened distribution. A new type of orbit appears for the angular-momentum stars that interact with the spherical potential of the nucleus, and "lose" their third integral (Martinet 1974). These chaotic orbits may wander virtually everywhere inside their permitted energy envelopes, even if started in the disk with low vertical velocities.

TABLE I. Mass model data.

Component	disk-halo	bulge	nucleus	dark halo
Mass ((km s^{-1})^2 kpc)	6.34×10^5	2.0×10^5	4.0×10^4	3.205×10^6
β_1	0.4	0	0	0
β_2	0.5	0	0	0
β_3	0.1	0	0	0
h_1 (kpc)	0.325	0	0	0
h_2 (kpc)	0.090	0	0	0
h_3 (kpc)	0.125	0	0	0
a (kpc)	3.0	0	0	0
b (kpc)	8.0	3.0	0.25	35

$M = M_{\rm dh} + M_{\rm n} + M_{\rm h} = 9.484\times10^{11}\,\mathscr{M}_\odot$

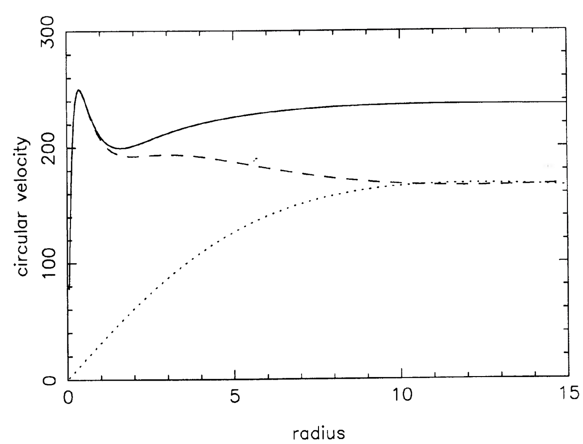

FIG. 1. The rotation curve of the galactic model is shown as the solid line. The dashed line is the contribution from the spherical components—the nucleus, bulge, and dark halo. The dotted line is the contribution from the disk-halo potential.

For the purposes of this paper we want to know the relative probability of finding stars of differing tangential velocities in a solar-neighborhood sample. To do this, representative orbits are integrated in the model potential given above. True halo stars already have large vertical velocities, and their low-angular-momentum members are not particularly notable in their vertical distribution. The orbits of interest are those that have modest radial and vertical velocities, and a range of angular momenta near zero. For radial and vertical velocities small compared to the circular velocity, less than, say, 50 km s^{-1}, the particular values chosen for these quantities do not affect the character of the orbit much. Chaotic orbits are a consequence of low angular momentum allowing the orbit to pass near the nucleus.

As the indicator of the chance of finding a star in a local sample, we calculate the mean vertical height as a function of the initial tangential velocity of the star. All these orbits have been started at the Sun's radius, with an outward radial velocity U of 44 km s^{-1} and a vertical velocity W of 30 km s^{-1}. The precise values chosen for U and W are irrelevant, provided that both the radial and tangential velocities are sufficiently low to keep high-angular-momentum stars in a disk distribution, less than about 30% of the circular velocity. Figure 2 shows the mean height distribution as a function of the initial tangential velocity measured against the circular velocity. The mean heights are averages taken over 10 Gyr in the model, integrated so as to begin and end at the plane, that typically gives 200 crossings. It is clear from Fig. 2 that stars having tangential velocities within 40 km s^{-1} of the zero-angular-momentum orbit to very large scale heights in this potential. The ragged nature of the height distribution reflects the complex properties of the low-angular-momentum orbits.

It is possible that some of the rise in the inferred rotation curve near the center of the galaxy is a result of a nonaxisymmetric nuclear mass distribution (Gerhard and Vietri 1986). Figure 2 also shows the effect of lowering the nuclear mass by 25%, reducing the peak rotation velocity 10% below the observed value. The result is to reduce proportionately the velocity width and height of the mean height distribution. The velocity where the mean scale height begins to rise is a combination of two factors, the contrast between the

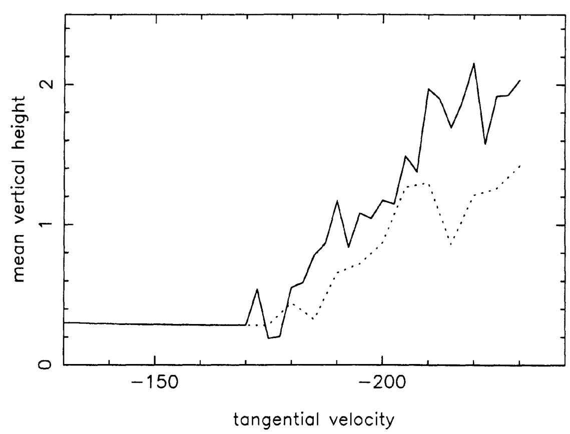

FIG. 2. Mean height above the $z = 0$ plane as a function of tangential velocity. The velocity is measured with respect to the circular orbit, at a velocity of -235 km s^{-1}. The solid line is for a full brass nucleus, and the dashed line shows the effect of reducing its mass by 25%.

bulk of the potential and the nuclear component, and the flattening of the disk-halo potential (see Binney 1982). The distance of closest approach to the center of the galaxy at the onset of chaotic behavior is around 1 kpc. The detailed shape of the nuclear region makes little difference to the classification of orbits that fall in from large radii. Orbits that miss the nuclear region can stay in a flattened distribution; those that enter the nuclear region have a large probability of being scattered to large heights. The details of the orbits are, of course, very sensitive to the details of the potential.

IV. TANGENTIAL VELOCITIES IN THE SOLAR NEIGHBORHOOD

The first question one might ask is whether there are likely to be any stars that are of any use for the application of this method, i.e., those that are simultaneously in a flattened distribution and possess a sufficiently large velocity dispersion or low enough rotation that significant numbers of stars have a low angular momentum. The two worst places to look would be the Population I stars, because they have too low a velocity dispersion and too high a rotation, and the extreme Population II stars, which have many low-angular-momentum stars, but the population as a whole does not appear to be very flattened, and would therefore show no removal of stars from a flat distribution. Ideally, one is after some transition component between the two populations. Whatever the origin, a useful kinematic population might be had in the high-velocity-dispersion, flattened components picked out by Gilmore (1984) and Zinn (1985). Below, we do find what can be considered an interesting, but not overwhelming, set of stars.

Accurate tangential space velocities (V) of stars are required to estimate the circular velocity with this method because the range of tangential velocity that has sufficiently low angular momentum to orbit out of the plane is fairly small, typically 50–80 km s^{-1} FWHM. The tangential velocities must be fairly precise, better than 10 km s^{-1}, in order that the region of missing stars not be washed out by velocity errors. Ideally, the sample has a known and well-defined kinematic bias, or no bias at all, so that any gap found can be assigned a statistical significance.

The primary data at our disposal are those found in the

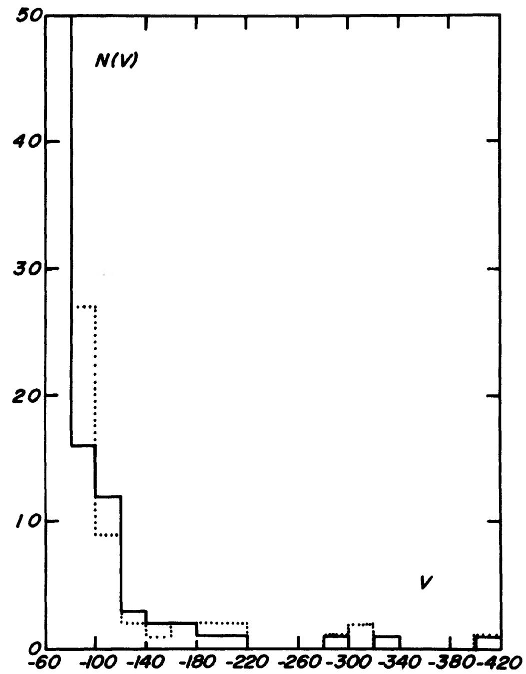

FIG. 3. Distribution of tangential velocities in the solar neighborhood found in the Wooley *et al.* (1970) and Gliese (1969) catalogs. The solid lines are Gliese's data.

catalog of Woolley *et al.* (1970), which is complete to a heliocentric distance of 25 pc. As such, it contains nearly all of the stars in the catalog of Gliese (1969), which extends to 20 pc. Figure 3 shows the distribution of the V velocities measured with respect to the Sun, for all the stars with $V < -60$ km s^{-1}, after combining the Gliese (1969) catalog and the Woolley *et al.* (1970) catalog. The data in the relevant range of velocities are listed in Table II. The number of stars with large tangential velocities is disappointingly small. Nevertheless, there is a complete absence of stars with V velocities in the range -220 to -280 km s^{-1}. We propose that this gap is the result of the action of the galactic nucleus discussed above.

Given the small number of stars, how significant is the gap? First, we assume that the stars are uniformly distributed in velocity. The ten stars in the 20 km s^{-1} bins between -180 and -300 km s^{-1} have a probability of $(4/7)^{10}$

TABLE II. Local high-velocity stars.

Bin start (km s^{-1})	Number
-140	3
-160	4
-180	3
-200	3
-220	0
-240	0
-260	0
-280	2
-300	2
-320	1
-340	0

$=0.0037$ of leaving the three bins from -220 to -260 km s^{-1} empty, or 0.018 of leaving any three adjacent bins empty. A similar estimate is provided by the Poisson distribution, dividing the interval from -160 to -340 km s^{-1} into three bins, each 60 km s^{-1} wide and, on average, containing five stars. The probability that at least one bin is empty is $1-(1-e^{-5})^3=0.020$. The assumption that all velocities are equally probable is unlikely, and a Gaussian distribution of velocities may be more appropriate. If we assume that the population in this range of velocities has a tangential velocity dispersion of 100 km s^{-1}, and is rotating at 35 km s^{-1} with respect to the local circular velocity, then for stars between -160 and -340 km s^{-1}, the intervals beginning at -160, -220, and -280 have probabilities of 0.681, 0.253, and 0.066 of being occupied. Clearly, this distribution may be a bit pessimistic since there are very few stars expected in the highest velocity range. Nevertheless, the probability in a sample of 15 stars of finding none in the specific range from -220 to -280 km s^{-1} is 0.012. In summary, the gap is interesting, although not overwhelming. With such small samples at our disposal, any probability estimate is at the mercy of small numbers. Our main point is that the data sets already available provide a substantial indication that this method for the determination of the circular velocity is relevant and practical as larger data sets become available.

We note that true halo stars are not excluded from this gap, so that some stars are expected in this velocity range. Some such stars can be found in Eggen's (1962, 1964) catalogs, but they are well beyond a heliocentric distance of 25 pc, and their space motions are uncertain. It may be possible to remove these stars on the basis of a metallicity index. The center of the gap in the current data set is at -250 km s^{-1} with respect to the Sun, or, correcting for the Sun's motion with respect to the LSR of $+15$ km s^{-1} leads us to suggest 235 km s^{-1} as the local circular velocity, with an estimated uncertainty of 10 km s^{-1}. Clearly, larger, better samples are required for a confident application of this method.

V. DISCUSSION

Although our method of determining the local circular velocity contains very few assumptions, the nearby stars contain impressively few members of much use for the method. The satellite *HIPPARCHOS* and new radial-velocity and proper-motion surveys permit a more confident estimate of V_0 to be made.

There are a few sources of systematic error that can bias the results somewhat. Our model orbits were done in an axisymmetric potential. If the center of the Galaxy is somewhat barred or triaxial, the details of the orbits will certainly be affected, but they will likely still be chaotic or quasichaotic. The only orbits affected will be the very low-angular-momentum ones that penetrate far into the central region, where they will arrive having large velocities. Hence the detailed shape of the nucleus should have relatively little effect on our estimate of V_0, although it could be modeled in detail.

Various random perturbations along the orbit, such as molecular clouds and their associated disturbances of surrounding material, should produce relatively little systematic error in V_0. In any case, the most important stars for this application have relatively high vertical and radial velocities at all times, and would be less susceptible to deflections than lower-velocity disk stars or gas. The most serious systematic error in the derived value of V_0 could come from a relatively open large-scale spiral pattern. The deviation of the vertex of the velocity ellipsoid (Delhaye 1965) requires that the potential be nonaxisymmetric. Fortunately, the effect is quite small for old disk stars, being a rotation of less than 10°. For a population with a characteristic radial-velocity dispersion in the solar neighborhood of 50–100 km s^{-1}, nonaxisymmetric effects probably continue to make an error of less than 5 km s^{-1}.

A particularly interesting object in the context of these low-angular-momentum orbits is one of the Sun's nearest neighbors, Kapteyn's Star. The best available data (Wooley *et al.* 1970) give its heliocentric (U,V,W) components as $(U,V,W)=(19,-288,-53)$ km s^{-1}. The orbit of this star is therefore very close to being chaotic, and quite sensitive to the details of the galactic potential. A further piece of intriguing information is Eggen's (1965) claim that Kapteyn's Star is the most prominent member of a moving group of stars, suggestive of an origin from a common association or cluster of stars. As this star is so close to zero angular momentum about the galactic axis, the stars that formed with it must have had very low relative velocities in order that they not be caught on exponentially diverging chaotic orbits. On the basis of the current data, we would favor an origin from the halo for Kapteyn's Star, and that it is not on a chaotic orbit. A few orbit integrations in our model indicate that the allowable initial velocity dispersion of any stars formed along with Kapteyn's Star that would keep them in its neighborhood is 2–3 km s^{-1}, not unreasonable for a small cluster of stars.

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