The Astronomical Unit Determined by Radar Reflections from Venus

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Radar reflections from the surface of the planet Venus at a wavelength of 12.5 cm yielded a value of the astronomical unit of 149,598,845±250 (p.e.) km, or a solar parallax of 8.7940976±147 based on an earth radius of 6,378,145 m. The computations were accomplished utilizing Doppler-frequency-shift and time-of-flight observations (range measurements) in conjunction with the “best” available planetary ephemerides of the earth and Venus. The investigations yielded proof of the transparency of the Venus atmosphere at 12.5 cm and some information on the radius of Venus. Systematic errors in the published ephemerides are also discussed.

I. INTRODUCTION

RADAR signals were reflected and successfully detected from the planet Venus from March 10 to May 10, or one month on either side of the 1961 inferior conjunction of April 11. The transmitting and receiving antennas were located at Goldstone, California, site of the NASA Deep Space Instrumentation Facility under the management of the California Institute of Technology’s Jet Propulsion Laboratory. The details of the operation of the experiment and a survey of the preliminary results can be found in Victor and Stevens (1961) and Muhleman (1961). The complete details of the instrumentation and techniques are described by Victor et al. (1961). The pertinent parameters of the radar installation are summarized in Table I.

The use of the radar observations for the determination of the astronomical unit is of primary concern here. Two somewhat different approaches to the problem are offered: first, the computation of the astronomical unit by a direct comparison of the observations with analogous quantities computed from the best available ephemerides; and second, the recomputation of the ephemerides and astronomical unit utilizing the radar observations and all available optical observations of Venus. This report is concerned with the first method, the latter technique being a vast project which can perhaps be accomplished successfully after several earth–Venus conjunctions have been observed radarmetrically.

Several types of observations were made during the JPL experiment which are, in a statistical sense, independent except for the velocity of propagation of the radio-frequency waves. However, an interdependence is introduced into the computation process by the use of “known” ephemerides. Three types of observations were used that are considered to be independent in the above sense: (1) the Doppler-frequency shift on the transmitter frequency due to the relative velocity of Venus with respect to the observing station, (2) the time of propagation of the signal to and from Venus as measured by an automatic closed-loop system, and (3) the time of flight measured by an open-loop radiometer technique. The Doppler frequency was detected in several somewhat different ways which cannot be considered independent. The details of these data types are described in Sec. II. The resulting astronomical unit determinations of all data types are in remarkable agreement and greatly add to the confidence in the results.

The ephemerides used in this work are essentially those given by Herget (1953, 1955) and Duncombe (1958). The first two are based primarily on the Newcomb tables of Venus and the sun; Duncombe (1958) applied a set of corrections to the mean orbital elements of the earth and Venus. Computations of the a.u. were made separately from the “Newcomb ephemerides” and the “Duncombe-corrected ephemerides,” as they will hereafter be called. This work yielded at least a partial verification of the Duncombe corrections.

II. INSTRUMENTATION

The Doppler shift of the Venus-reflected signal was measured with a narrow-band closed-loop receiver. The transmitted frequency was derived from an Atomichron, which is a crystal oscillator locked to a cesium resonance line. The frequency stability of this reference was probably better than 1 part in 10^10 over the signal transit time. The echoed signal was detected utilizing an automatic phase-tracking loop with a 5-cps bandwidth when the signal was at the receiver’s threshold. No free-running oscillators were employed in the receiver. The fundamental reference signal was that from the same Atomichron used in transmitting. Thus, the signal was detected with a stability of 1 part in 10^10, with a short-period jitter of about 1.5 cps rms due to the characteristics of the 5-cps tracking loop.
The frequency of the detected signal was determined in several ways. The primary frequency data were obtained by a zero-crossing counter operating for a period of 1 sec. The frequency was also counted by identical equipment for periods of 10 and 60 sec. The latter measures of frequency are termed “integrated Doppler” in this report and represent the change in the range over the period counted. In each case, the counters had a resolution of 1 cps, corresponding to an accuracy of 1 part in $10^3$ in the case of the 1-sec count and somewhat better for the longer counting intervals.

The closed-loop range measurements were made with an amplitude-modulation system. The transmitter was amplitude-modulated by a transmitter code which was a long, pseudo-random binary wave form. The returned signal was received by the ranging receiver, a type of correlation receiver for which the receiver coder forms the local model for correlation detection of the received code. When the received code was acquired—that is, when the local model was in phase with the received code—the phase difference between the transmitter code and the receiver code was a function of the propagation time to the planet and back. This propagation time is a measure of the range for a known propagation velocity. The ranging receiver automatically tracked the received code, and the phase-measuring device provided a continuous real-time measurement of range.

A single code length consisted of 255 digits, with a keying rate of 122 digits/sec. Thus, the fundamental code period was approximately 2.09 sec, or an increment of 626,000 km in the two-way range, which was easily sufficient to resolve the range ambiguity. Once the reference code was synchronized with the echoed code, the range was resolved to the above figure. The change in phase with time between the codes due to the Doppler shift from the planet was measured continuously, with a reference at 497.5 kc/sec, thus yielding a range resolution of 2.01 μsec, or about 603 m. Because the weak signal and the ambiguity of the range to the various points on the surface of Venus contribute to the reflected wave, an accuracy equal to this fine resolution could not be achieved in practice. The data indicate that the range measurements have a peak deviation of about 50 km. The details of the closed-loop range system are described by Easterling (1961).

The open-loop range measurements were obtained with a switched radiometer very similar to the conventional devices used in radio astronomy. However, in the case of the radar operations described here, it was possible to switch the transmitter on and off, as opposed to the conventional switching of the receiver from the signal source to a calibrated noise source. The range-measuring mode of operation used two channels of the switched radiometer. One channel was switched on and off in phase with the transmitter switching, and the other channel was switched at the midpoint of the transmitter-switching period. The transmitter was switched with a square-wave generator at rates of $\frac{1}{2}$, 1, 2, 4, 8, 16, and 32 cps. The relative outputs of the two channels were read from recordings on strip charts. The orbital motion of Venus relative to the earth showed up clearly as the channel outputs traced a triangular wave. A precise time-of-flight measurement was made by noting the time at which the triangular wave crossed zero. At that instant, an integral number of wavelengths at the keying frequency filled the space between the transmitter, the surface of Venus, and the receiver. The time of zero-crossing was determined graphically with a precision of about 200 μsec, corresponding to about 60 km in round-trip range. The details of the instrumentation are described by Goldstein (1961).

### III. Relativistic Doppler Equation and Propagation Time

The Doppler-frequency shift on the radio wave must be computed from the ephemerides with precision for comparison with observations. The computation of velocity from the ephemerides is discussed in Sec. IV. Here we are concerned with the conversion of the velocities to Doppler frequency. The Doppler shift is a function of (1) the velocity of the center of mass of Venus at the instant the wave front strikes the surface of the planet with respect to the position and velocity of the transmitting station and (2) the velocity and position of the receiving station at the instant the reflected wave front reaches the receiving station with respect to the velocity of the center of mass of Venus at the instant of reflection. It is important to consider the velocity of the earth station at the instant of reception with respect to the velocity of the earth station at the instant of transmission, as much as the earth’s rotation and acceleration during the time required for the signal to traverse the path affects the Doppler measurements. It is possible to solve this problem without considering the exact mechanism of reflection from the surface of Venus. It is assumed that, from a heliocentric viewpoint, the ray is taken between the points in space occupied by the center of Venus at the time of reflection to the position of the receiver at the instant of reception. Thus, the effect of the rotation of Venus about its axis is assumed to be zero. In practice, the returned signal spectrum has been broadened by the rotation of Venus; but the receiving system responds to the center frequency, which is apparently the correct measure of the motion of the center of mass. With the above assumptions, the use of the law of reflection in the inertial frame fixed at Venus and, consequently, consideration of angular aberrations, are avoided.

An inertial frame is constructed coincident with the transmitter at the instant of transmission, with the $x$-coordinate axis parallel to the direction of the relative velocity vector of Venus at the time of reflection with respect to velocity vector of the earth station at the...
time of transmission. All positions and velocities are
the geometrical quantities taken from the heliocentric
ephemerides. It then follows that the ray from the
transmitter to Venus is in the same direction as given
by the ephemerides. If the angle between the ray and
the relative velocity vector (the x axis) is called \(\alpha_0\),
the frequency seen by the observer on Venus, \(v'\), is found
from the standard Lorentz transformation to be

\[
v' = v\gamma_1(1 - \beta_1 \cos \alpha_0),
\]

where \(v = \) transmitted frequency, \(\gamma_1 = (1 - \beta_1^2)^{-\frac{1}{2}}, \beta_1 = \frac{v_1}{c}, v_1 = \) the magnitude of the relative velocity of
Venus at the instant of reflection with respect to the
earth station at time of transmission. It is then assumed
that the observer on Venus retransmits a spherical
wave at the frequency \(v'\). In the time elapsed while
the wave front traverses the path, the receiving station
has acquired a new velocity vector relative to Venus as
a result of the integrated acceleration of the earth and
a change in direction of the component of velocity.
The latter effect is due to the rotation rate of the earth.
Another inertial frame is selected at the receiver at
the moment of reception which is parallel to the relative
velocity vector of Venus at reflection with the velocity
vector of the earth station at reception. The ray that
is received is directed from the earth station at this
epoch to the position of Venus at reflection, as given
by the ephemerides. This ray makes an angle of \((\alpha_0 + \epsilon)\)
with the relative velocity vector, where \(\epsilon\) is very small.
Thus, if the ray is thought of as being transmitted
backwards in time and space at the frequency of final
reception \(\bar{v}\), the observer on Venus would see the
frequency \(v'\), requiring the following relationship to be
true:

\[
n' = \gamma_2[1 + \beta_2 \cos(\alpha_0 + \epsilon)],
\]

where \(\gamma_2 = (1 - \beta_2^2)^{-\frac{1}{2}}, \beta_2 = \frac{v_2}{c}, v_2 = \) the magnitude of the relative velocity of Venus at the instant of reflection
with respect to the earth station at the instant of
reception.

One may now solve for the received signal frequency
\(\bar{v}\), with Eqs. (1) and (2) yielding

\[
\bar{v} = \frac{v_1}{\gamma_2[1 + \beta_2 \cos(\alpha_0 + \epsilon)]},
\]

The quantities \(\beta_1 \cos \alpha_0\) and \(\beta_2 \cos(\alpha_0 + \epsilon)\) are the
projections of the relative velocity vectors in the direction
of the rays and, consequently, are equal to the rate of
change of the distance over the transmitted and
reflected paths. Therefore,

\[
\bar{R}_{12} = c\beta_1 \cos \alpha_0
\]

and

\[
\bar{R}_{23} = c\beta_2 \cos(\alpha_0 + \epsilon).
\]

Both \(\bar{R}_{12}\) and \(\bar{R}_{23}\) were computed directly from the
ephemerides using the positions of the earth and Venus
at the various epochs as described in Sec. IV. Equation

(3) becomes

\[
\bar{v} = \frac{\gamma_1}{\gamma_2} \frac{1 - \bar{R}_{12}/c}{1 + \bar{R}_{23}/c},
\]

The receiving system measures the difference \((\bar{v} - v)\),
and this quantity must be computed from the ephemerides.
Experimentation with the computation shows that terms of order \(\beta^2\) must be retained in an expansion
of Eq. (6) to be consistent with the quality of the
measurements. The term \((\gamma_1/\gamma_2)\) will be examined
separately:

\[
\gamma_1 = \left(1 - \beta_1^2\right)^{-\frac{1}{2}}, \gamma_2 = \left(1 - \beta_2^2\right)^{-\frac{1}{2}},
\]

\[
\approx \left[\left(1 - \beta_1^2\right)\left(1 + \beta_2^2\right)\right]^{\frac{1}{2}}
\]

\[
\approx \left[1 - \left(\beta_2^2 - \beta_1^2\right) - \beta_2^2 \beta_1^2\right]^{\frac{1}{2}}.
\]

Write

\[
\beta_2 = \beta_1 + \delta/c,
\]

where \(\delta\) is the difference in the relative velocities over
the time of the signal transit, amounting to about 10
m/sec in the worst case.

\[
\gamma_1/\gamma_2 \approx 1 - \beta_1(\delta/c) = 1 \text{ to a term of about } 10^{-11}.
\]

The expansion of Eq. (6) becomes

\[
\bar{v} = \frac{\left(1 - \frac{\bar{R}_{12}}{c} - \frac{\bar{R}_{23}}{c^2} - \frac{\bar{R}_{23}^2}{c^2}\right)}{\gamma_2}
\]

(7)

\[
(\bar{v} - v) = -\frac{\bar{R}_{12}}{c} \frac{\bar{R}_{23}^2}{c^2} - \frac{\bar{R}_{12} \bar{R}_{23}}{c^2} - \frac{\bar{R}_{23}^2}{c^2}.
\]

Equation (7) was adopted for all of the subsequent
calculations.

A careful consideration of the general relativistic
effects shows them to be negligible. The primary effect
is the shift in the frequency of the signal due to changes
in gravitational potential (Möller 1952). The effect on
the radio signal is an increase in frequency as the wave
front travels to Venus caused by the increase in the
sun's gravitational potential. At the experimental
frequency, this shift is approximately 6 cps. However,
a decrease in frequency of almost the exact magnitude
occurs after reflection as the wave returns to the earth.
The only residual effect is that due to the change of
the position of the earth relative to the sun. Further,
the reference oscillator fixed to the earth has experienced
the same change in gravitational potential, and hence,
no effect could be detected.

The propagation time for the signal depends upon
the distance from the transmitter at the instant of
transmission to the near surface of Venus at the time
of reflection, plus the distance from Venus at reflection
to the receiver at the time of reception. The reference
epoch \(\delta\) in each case was taken to be the instant of
reflection; i.e., at this epoch (expressed in Universal
Time), the exact frequency of the wave at the receiving
antenna had to be computed from the ephemerides.
Similarly, the propagation time for the wave arriving
at the receiver at \( t_0 \) was needed. Thus, starting with a
given epoch \( t_0 \), the coordinates and velocity of the
earth station were taken from the ephemerides as the
initial conditions. An estimated time (before \( t_0 \)), at
which the wave had been reflected from Venus, based
on the geometric distance at \( t_0 \), was then used to com-
pute the first estimate of the coordinates of Venus at
reflection. An iterative procedure was employed to
correct the Venus-receiver propagation time using a
corrected position of Venus. Once the time of propa-
gation from Venus to the receiver and the position of
Venus were found, it was possible to compute the
velocity of Venus for the Doppler shift, using \( t_0 \) minus
the Venus-receiver propagation time as the argument.
To complete the solution, a similar procedure was then
used to find the position in space and time of the earth
at the time of transmission; of course, the argument
used for the planetary coordinates was Ephemeris Time,
assumed to differ by 35 sec from Universal Time. The
entire calculation was repeated for each epoch of each
observation. In all cases, the vacuum value of the
velocity of light was used, since dispersion and refraction
effects through the earth's atmosphere, the inter-
planetary medium, and any reasonable model of the
Venus atmosphere at 2388 Mc can easily be shown to
be negligible. It was assumed that the reflections
occurred at the hard surface of Venus, an assumption
which will be justified in Sec. VI.

IV. COMPUTATION OF POSITIONS AND VELOCITIES
FROM EPHEMERIDES

In the "Tables of the Sun" and "Tables of Venus,"
Newcomb (1898a and b) presents for each body the
numerical values for his first-order general perturbation
theory. Using the numerical tables generated from the
theory, one may compute the heliocentric ecliptic co-
ordinates of the earth-moon barycenter, the earth, and
Venus. Newcomb introduced errors of certain types in
the preparation of the numerical tables:

1. The omission of certain perturbation terms that
were considered negligible.

2. The use of incorrect coefficients in certain
perturbation terms.

3. The use of inaccurate values for certain terms
in the expressions for the principal (classical)
elements of the unperturbed orbit.

Clemence (1942) has described errors of all three types
which occur in the Venus perturbations upon the earth.
All of the errors mentioned by Clemence amount to
about 0'557 in longitude.

Errors of type 1 in the perturbation terms arise from
the fact that no numerical tables were prepared for the
effects of Mars and Saturn, thus introducing an error
in latitude of the earth as large as 0'037.

In the "Tables of the Sun," Newcomb (1898a) used
an expression of the form

\[ a + bT + c \sin(d + eT) \]

for the mean anomaly of Jupiter, the effect of which is
included by means of argument III (argument III is
the mean anomaly of Jupiter when the earth was at its
last perihelion). Although he gives numerical values
for \( a \) and \( b \) in Table G (Newcomb 1898, p. 21), no
values appear for the constants \( c, d, \) or \( e \). However, in
his "Tables of Mars" (Newcomb 1898c), he gives values
used for the mean anomaly of Jupiter in computing
perturbations on Mars. One is unable to reconcile the
results obtained using these values with the tabulated
values of argument III in the numerical tables. A
statistical analysis shows that the values given for \( a \)
in Table G (Newcomb 1898a) needed to be increased by
\(+0.052\) to yield results consistent with the tabulated
argument III.

Herget, working from Newcomb's numerical tables,
computed ephemerides of the earth-moon barycenter,
the earth (Herget 1953), and Venus (Herget 1955). In
the introduction, Herget (1953) states that the
published coordinates should differ from those obtained
from Newcomb's numerical "Tables of the Sun" only
in one place; namely, that the tabular centennial
motion of the perihelion should receive a correction
amounting to \(-478\). Clemence (1947) has stated that
this correction is most easily effected by introducing a
correction of \(+0.0013\) \( T \) to the argument \( M \).

The actual values used in the analysis of the Venus
radar data were computed with a "tracking" program
written for the IBM 7090 computer. The coordinates
to be smoothed were obtained directly from Newcomb's
theory, rather than using Herget's published results,
and contained corrections to the known errors in the
numerical tables. An \( n \)-body numerical integration,
starting with "injection" position and velocity, was
compared with the coordinates written on the magnetic
tape by the Newcomb program, and corrections to the
injection conditions were derived using a least-squares
iterative procedure. Several iterations yielded the best
injection values over a 120-day arc, which minimized the
standard deviation of the position residuals. These
residuals were reduced to a few parts in \( 10^6 \), which was
consistent with the roundoff in the tabulated data.
Velocity data were obtained at each epoch of interest
as a consequence of the Runge-Kutta numerical integra-
tion procedure. The velocities obtained in this
manner are smooth to seven figures and probably
accurate to a few parts in \( 10^6 \). The ephemerides ob-
tained with the above technique are considered a
smooth equivalent to the numerical tables of Newcomb,
including only the change in the argument \( M \) referred
to above. Subsequently in this report, the ephemerides
will be referred to as the "Newcomb ephemerides."

Corrections to the mean elements of the orbits of the
earth and Venus were computed by Duncombe (1958)
with a least-squares analysis of several decades of
ASTRONOMICAL UNIT FROM VENUS RADAR

Fig. 1. Difference between Doppler velocity observations and theoretical Doppler curve generated using a.u. computed from observations.

Venus observations and are restricted to corrections in the angular positions of the bodies in the form of a constant plus a term varying linearly in time; i.e., \( a + bT \). The primary effect of the corrections is to move the position of Venus “forward” in its orbit relative to the earth at the time of interest. A smaller radial correction to the orbits of the two bodies was also obtained. The resulting corrections, as reported by Duncombe (1958), are:

for the earth:
\[
\Delta e_\oplus = -0^\circ 10 + 0^\circ 00 \ T, \\
\Delta e = +0^\circ 04 - 0^\circ 29 \ T, \\
\Delta L_\oplus = -0^\circ 39 + 0^\circ 45 \ T;
\]

for Venus:
\[
\Delta e_\oplus = +0^\circ 10 + 0^\circ 53 \ T, \\
\Delta e = -0^\circ 12 + 0^\circ 01 \ T, \\
e_\oplus \Delta \pi = +1^\circ 01 - 0^\circ 04 \ T, \\
\Delta i_\oplus = +0^\circ 08 - 0^\circ 02 \ T, \\
\sin i_\oplus \Delta \Omega_\oplus = +0^\circ 21 + 0^\circ 02 \ T.
\]

The corrections actually used in these computations are by Duncombe (1961):

for the earth:
\[
\Delta e_\oplus = -0^\circ 113 \ T, \\
\Delta e = +0^\circ 045 - 0^\circ 29 \ T, \\
\Delta M_\oplus = +4^\circ 78 \ T;
\]

for Venus:

Same as those of Duncombe (1958) shown above.

The second set of corrections is essentially the same as the first but is somewhat improved.

The Duncombe corrections were incorporated in the program which evaluated the Newcomb theory, and a new magnetic tape was written which served as input to the tracking program. The resulting smoothed ephemerides for both the earth and Venus are called the “Duncombe-corrected ephemerides.”

V. ASTRONOMICAL UNIT COMPUTATIONS

The actual mechanics of the astronomical unit computations were accomplished using an iterative least-squares procedure. All of the Doppler observations were available on punched paper tape and were easily converted to IBM punched cards. The range numbers were available on printed adding-machine tape and were punched onto IBM cards by hand. The cards for all of the observables were written on magnetic tape for processing by the IBM 7090 computer. The data, available at 10-sec intervals for periods of a few hours a day, were plotted from the magnetic tape for the purpose of editing out wild points. The plotting process was felt to be more reliable than an automatic-rejection scheme in the computer; a new magnetic tape was prepared from the edited deck of cards. The difference between the observations and the ephemeris values using the final astronomical unit estimates was also plotted and inspected closely. Typical plots of this type are shown in Figs. 1, 2, and 3.

The correction to the ith estimate of the a.u., \( \delta A_i \), was computed with the relationship

\[
\delta A_i = \frac{\sum_j \left( \frac{\partial \xi_j}{\partial A_i} \right) \delta \xi_j}{\sum_j \left( \frac{\partial \xi_j}{\partial A_i} \right)^2}, \quad (8)
\]

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where $\xi$ is any one of the observables: Doppler frequency, Doppler cycles counted over 10 or 60 sec, or propagation time. The partial derivatives, $\partial \xi / \partial A_i$, are the changes in the observables with respect to a small change in the $i$th estimate of the a.u., and $\delta \xi_j$ is the observation minus the computed value, using the $i$th estimate of the a.u. at the $j$th time index. Three or four iterations were normally required to converge to a final value of the a.u.

The standard deviation of the residuals was a function of the signal strength and, consequently, a function of the separation distance between Venus and earth. Near conjunction, the standard deviation of the Doppler velocity residuals was about 0.14 m/sec and that of the range residuals about 40 km. Each value of the a.u. computed from the Doppler observations used an estimated average of about 100 independent samples. On the average, about 1500 time-of-flight observations from the closed-loop ranging system were used for each value of the a.u. computed from these data, but the sinusoidal nature of the time-of-flight residuals (see Fig. 2) made it difficult to estimate the number of independent samples. The systematic nature of these residuals is due to the response characteristics of the closed-loop ranging system.

A value for the a.u. was computed from each continuous run of observations for each data type. Because of the nature of the receiving system, data of

**Fig. 2.** Difference between closed-loop range observations and theoretical range curve generated using a.u. computed from observations.

**Fig. 3.** Difference between integrated Doppler observations and theoretical integrated Doppler curve generated using a.u. computed from observations.
one type only could be taken at any given time, with the exception of the Doppler frequency and the Doppler counted over 10 or 60 sec, which were measured simultaneously. The only observations that were made before conjunction were the Doppler frequencies. The resulting values for the a.u. are shown in Fig. 4, where the open circles show results using the "Newcomb ephemerides" and the closed circles show results using the "Duncombe-corrected ephemerides." The results are seen to diverge downward as conjunction is approached and return from above after conjunction, approaching a stable value toward the western elongation. For an unknown reason, the quality of the results before conjunction is much superior to that after conjunction. The effect of the Duncombe corrections was to

1. Raise the a.u. near the eastern elongation (Mar. 23) by 1200 km.
2. Raise the a.u. near the eastern side of conjunction (Apr. 7) by 6900 km.
3. Lower the a.u. near the western side of conjunction (Apr. 13) by 8900 km.
4. Lower the a.u. near the western elongation (May 3) by 400 km.

Thus, the effect of the corrections was to "straighten" the curve as well as to cause the result nearest the eastern and western elongations to approach agreement. A detailed explanation of this effect is given in Sec. VI.

The results from the closed-loop ranging system observations are shown in Fig. 5. A small upward trend may be seen in the data, whose slope is reduced by the Duncombe corrections. The effect of the corrections was to

1. Lower the a.u. on Apr. 16 by 166 km.
2. Lower the a.u. on May 5 by 346 km.

The results from the open-loop ranging system observations are presented in Fig. 6. The values obtained using the "Newcomb ephemerides" were found to be remarkably constant, with a standard deviation from the mean of 40.7 km. The effect of the Duncombe corrections was, of course, to introduce a downward trend to the results similar to that of the closed-loop ranging system. This is contradictory to the previous results, in which the corrections always tended toward a nonvarying result. No explanation of this effect is
available, but it should be noted that the graphic reduction of the open-loop ranging data was made with the "Newcomb ephemerides" as working ephemerides. The method of data reduction has been closely examined for erroneous procedures that might lead to a constant result.

The results of the Doppler counted over 10 and 60 sec are presented in Fig. 7. The errors on these results are completely correlated with the errors on the Doppler frequency results after conjunction, but they are about 700 km higher.

From the nature of the figures presented here, and from the character of the errors removed by Duncombe's correction, it is clear that the results from Doppler measurements are most accurate near the elongation points and those from the range measurements at conjunction. Extrapolation of the computations to these points (where necessary) leads to the summary presented in Table II. The uncertainty shown in case I of Table II is the estimated probable error in the extrapolation of the data toward the eastern elongation. It should be noted from the data presented in Table II that the random error among the pre-conjunction determinations for the Doppler is a few km. However, the ephemerides resulting from planetary orbit theory are not sufficiently good for this small error to be used to advantage. The uncertainties shown in Table II for cases II through V are essentially rms deviations of the scatter. The results of Table II do not indicate any systematic errors.

A combination of the results of Table II with several weights is shown in Table III. The uncertainties of Table II were interpreted as standard deviations for

<table>
<thead>
<tr>
<th>Method</th>
<th>a.u., km</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arithmetic mean</td>
<td>149 598 754±340*</td>
</tr>
<tr>
<td>Weighted mean using reciprocal standard deviations</td>
<td>149 598 845±180</td>
</tr>
<tr>
<td>Weighted mean using reciprocal variances</td>
<td>149 598 884±126</td>
</tr>
</tbody>
</table>

* Uncertainties combined from Table II using the same weighting system as the corresponding value of the a.u.; i.e., rms, etc.

Table II. Summary of results.

<table>
<thead>
<tr>
<th>Case</th>
<th>Description</th>
<th>&quot;Newcomb&quot; a.u., km</th>
<th>&quot;Duncombe&quot; a.u., km</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Doppler frequency near eastern</td>
<td>149 597 550±200</td>
<td>149 598 950±200</td>
</tr>
<tr>
<td></td>
<td>elongation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>Doppler frequency near western</td>
<td>149 599 650±500</td>
<td>149 598 250±500</td>
</tr>
<tr>
<td></td>
<td>elongation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>Open-loop range at conjunction</td>
<td>149 598 970±100</td>
<td>149 598 930±100</td>
</tr>
<tr>
<td>IV</td>
<td>Closed-loop range at conjunction</td>
<td>149 599 150±100</td>
<td>149 598 850±100</td>
</tr>
<tr>
<td>V</td>
<td>Long-count Doppler near western</td>
<td>149 599 750±500</td>
<td>149 598 750±500</td>
</tr>
<tr>
<td></td>
<td>elongation</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 6. Value of a.u. computed for each set of open-loop range observations.

Fig. 7. Value of a.u. computed for each set of integrated Doppler velocity observations.
the calculations of Table III. The value of 149 598 700 ±180 km was selected as the most likely result, based on the facts and a strong intuitive understanding of the entire experiment. However, because of systematic errors, which are discussed in detail in Sec. VI, it is felt that the probable error should be set at ±250 km.

VI. ERROR ANALYSIS

Several types of errors are quite possible in this analysis as a result of (1) uncertainties in the theory of the planetary motion, (2) systematic errors in extracting numerical quantities from the ephemerides, (3) the uncertainty in the vacuum speed of light, (4) dispersion effects on the speed of light, (5) the uncertainty of the radius of Venus, and (5) equipment biases and frequency drifts.

The simplified analysis presented below offers much insight into the question of the planetary theory employed in the construction of the ephemerides used here. The procedure, though essentially two-dimensional, explains nearly completely the wide variation in the results presented in this report.

The distance between the earth and Venus at any time can be expressed in terms of the orbital elements from the law of cosines (see Fig. 8):

\[ R^2 = R_\oplus^2 + R_\odot^2 - 2R_\oplus R_\odot \cos \alpha, \]

where \( \alpha \) is the angle at the sun between the radius vector to Venus \( R_\odot \) and the radius vector to the earth \( R_\oplus \) and is given by

\[ \cos \alpha = \cos (l_\odot - l_\oplus) \cos (L_\odot - L_\oplus) \]

\[ + \sin (l_\oplus - l_\odot) \sin (L_\odot - L_\oplus) \cos i_\odot, \]

(10)

where

\[ L_\oplus = \pi/2 + v_\oplus \]

(11)

and

\[ l_\odot = \Omega_\odot + \omega_\odot + l_\oplus. \]

(12)

The inclination of the Venus orbit to the earth orbit \( i_\oplus = 3^\circ 23'37''07 \), and thus, \( \cos i_\odot \) is very nearly unity. Hence, the orbits are approximately coplanar, and Eq. (10) is nearly

\[ \cos \alpha \approx \cos (l_\odot - L_\oplus). \]

(13)

The true longitudes of the two planets are closely coupled because of the low inclination of Venus, and, consequently, the separation of errors into earth terms and Venus terms is difficult. Thus, the primary effect of the Duncombe corrections is essentially the sum of \( \Delta L_\oplus \) and \( \Delta l_\odot \), which, for \( T=0.61 \), gives

\[ \delta (l_\odot - L_\oplus) = +0^\circ 54. \]

That is, Venus is advanced by approximately 0.54 sec of arc relative to the earth.

The effect of errors of this type on range and Doppler velocity observations can be studied by varying \( l_\oplus - L_\oplus \) in Eq. (9). If the approximations \( \cos i_\odot = 1 \) and \( R_\odot = R_\oplus = 0 \) are used, the effect of an error, \( \delta (l_\odot - L_\oplus) \), on the astronomical unit determination using range and Doppler velocity can easily be found. The resulting equation for range data is found to be

\[ \delta A_\oplus = A_\oplus (R_\odot R_\oplus / R^3) \sin (l_\oplus - L_\oplus) \delta (l_\oplus - L_\oplus). \]

(14)

The corresponding Doppler velocity equation is

\[ \delta A_\oplus = A_\oplus \cos (l_\oplus - L_\oplus) \delta (l_\oplus - L_\oplus), \]

(15)

where \( A_\oplus \) is the astronomical unit.

In Fig. 9, Eq. (14) has been plotted for values of \( \delta (l_\odot - L_\oplus) = 0^\circ 2, 0^\circ 4, \) and \( 1^\circ 0 \), where a value of \( A_\oplus = 149 598 700 \) was assumed. A comparison of the results for the closed-loop ranging system in Fig. 5 with

![Fig. 9. Effect of ephemeris errors on a.u. computed from range observations.](image-url)
Fig. 9 indicates that an error of about 1°0 exists in the "Newcomb ephemerides" and that the effect of the corrections was to remove about half the error, consistent with the value of $\delta (l_o - L_o) \approx 0.54$ reported above.

Equation (15) has been plotted for values of $\delta (l_o - L_o) = 0.1, 0.2, 0.4, 0.8$, and 1.0 and is shown in Fig. 10. Again, a comparison of Fig. 4 with Fig. 10 indicates an error in the "Newcomb ephemerides" of about 1 sec of arc, and the Duncombe corrections are apparently about half the change required.

Several important conclusions may be drawn from the data and the simplified theory presented above:

1. The measurement of the range of Venus with the accuracy of the current observations yields a value for the astronomical unit which is insensitive to errors in the ephemerides.
2. The measurement of the Doppler velocity with the accuracy of the current observations yields values for the astronomical unit that are quite sensitive to errors in the ephemerides.
3. Because of conclusions 1 and 2, the Doppler velocity data are very valuable for the computation of corrections to the ephemerides.
4. Because of the coupling of the longitudinal motion of both the planets, it is difficult to correct the elements of the two planets separately with radar observations at one Venus conjunction.

Elaboration of conclusion 4 is appropriate. It would be possible to compute corrections to the elements of Venus and the earth from the radar observations that would yield nearly constant values of the astronomical unit. However, this procedure would yield corrections to the orbits which would probably not apply over the entire orbits of Venus and the earth and could therefore result in an erroneous determination of the astronomical unit. Consequently, further work will be done to complete the computations.

<table>
<thead>
<tr>
<th>Constant</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum velocity of light</td>
<td>299 793.0 km/sec</td>
</tr>
<tr>
<td>Rotational rate of the earth</td>
<td>0.00417807416 deg/sec</td>
</tr>
<tr>
<td>Radius of Venus</td>
<td>6100 km</td>
</tr>
<tr>
<td>Coordinates of the receiving antenna</td>
<td></td>
</tr>
<tr>
<td>Distance from earth center</td>
<td>6372.0355 km</td>
</tr>
<tr>
<td>Geocentric latitude</td>
<td>35.260619 deg</td>
</tr>
<tr>
<td>Longitude</td>
<td>243.151750 deg</td>
</tr>
<tr>
<td>Coordinates of the transmitting antenna</td>
<td></td>
</tr>
<tr>
<td>Distance from earth center</td>
<td>6372.0362 km</td>
</tr>
<tr>
<td>Geocentric latitude</td>
<td>35.119983 deg</td>
</tr>
<tr>
<td>Longitude</td>
<td>243.195194 deg</td>
</tr>
<tr>
<td>Earth–moon mass ratio</td>
<td>81.450</td>
</tr>
<tr>
<td>Reciprocal mass of the earth</td>
<td>$3.33432 \times 10^6$</td>
</tr>
<tr>
<td>Ratio of the a.u. to the earth radius for the lunar ephemerides</td>
<td>23 438.6</td>
</tr>
<tr>
<td>Planetary reciprocal mass ratios</td>
<td></td>
</tr>
<tr>
<td>Mercury</td>
<td>$6 \times 10^6$</td>
</tr>
<tr>
<td>Venus</td>
<td>$4.08 \times 10^6$</td>
</tr>
<tr>
<td>Earth–moon</td>
<td>$3.2939 \times 10^6$</td>
</tr>
<tr>
<td>Mars</td>
<td>$3.0835 \times 10^6$</td>
</tr>
<tr>
<td>Jupiter</td>
<td>$1.04735 \times 10^6$</td>
</tr>
<tr>
<td>Saturn</td>
<td>$3.5016 \times 10^6$</td>
</tr>
<tr>
<td>Uranus</td>
<td>$2.2809 \times 10^6$</td>
</tr>
<tr>
<td>Neptune</td>
<td>$1.9314 \times 10^6$</td>
</tr>
<tr>
<td>Pluto</td>
<td>$3.6 \times 10^6$</td>
</tr>
<tr>
<td>Ephemeris Time–Universal Time</td>
<td>35 sec</td>
</tr>
</tbody>
</table>
bine the normal equations resulting from Duncombe’s work (1958) with the similar equations derived from the radar observations. This will be a considerable task. The ultimate integrity of the computation can be assured only after radar observations of several conjunctions have been analyzed.

The planetary ephemerides contain the coordinates of the planets to seven significant figures in astronomical units. The coordinates of the earth and of Venus were fit with a least-squares procedure, utilizing the physical constants of Table IV. The least-squares fit over arcs of about 100 days yielded maximum residuals of about 2×10⁻⁷ a.u. Arcs greater than 100 days caused some difficulties, and consequently, work on this project will continue. The velocities obtained by this procedure are probably accurate to a few parts in 10⁶. The possibility of a systematic error at this point should be fully realized. However, the agreement in the results between the range data and the velocity data suggests that this error is not serious.

The vacuum speed of light that was used in these computations is 299,793.0±0.3 km/sec, as given by Froome (Bergrstrand 1956). The uncertainty of ±0.3 km/sec reported by Froome corresponds to an uncertainty in the a.u. of ±150 km, which fixes a lower bound on the accuracy of this work. However, it is of interest to examine carefully the use of a vacuum velocity for this work; i.e., to ignore the effects of angular refraction and dispersion.

The effect of angular refraction in the atmospheres of the earth and Venus is to increase the path length as a result of the curvature of the ray. The magnitude of this effect for any reasonable atmosphere must be a small fraction of the total thickness of the atmosphere in the direction of the ray. In the present experiment, the data were taken from the zenith to within 10 deg of the horizon. If the portion of the earth’s atmosphere which produces significant angular refraction is assumed to be 10 km the maximum path length would be about 60 km; consequently, the error associated with that path would be negligible with respect to the uncertainty in the vacuum speed of light. In the case of Venus, it is clear from the experimental evidence that most of the returned signal was reflected from an area within a few degrees of the near point of the surface of the earth. Consequently, the path of the rays that combined to make the reflected signal was normal to the Venussian atmosphere and hence produced no appreciable angular refraction.

Considerations of dispersion in the Venussian atmosphere (or ionosphere) are considerably more difficult and important than the angular refraction. Only vastly differing theoretical models exist for the regions above the surface of Venus. Here, a model of a uniform distribution of electrons is adopted for purposes of discussion. Such a model is the only reasonable one that could seriously affect the propagation time of a signal at the experimental frequency.

The index of refraction for the group velocity \( n_g \) is defined by the relation

\[
{\frac{c_0}{v_g}} = n_g.
\]

Then, the group index of refraction is related to the index of refraction by the equation

\[
{\frac{n_g}{n}} = 1 + f(\frac{dn}{df}).
\]

If it is assumed that there is no magnetic field in the medium and that the collision of the particles comprising the medium may be neglected, the index of refraction is given to good accuracy by the following approximation to the Appleton–Hartree equation (Ratcliffe 1959)

\[
\sigma^2 = 1 - (f_0/f)^n,
\]

where \( f_0 \) is the critical frequency of the medium, which, for the case of free electrons, is proportional to the square root of the electron-number density \( N_e \). The primary effect of collisions is to cause absorption of the signal power and will be considered below, whereas the neglect of the Venussian magnetic field, if it exists, is primarily the neglect of polarization phenomena which are of no concern here.

These effects in the earth’s ionosphere are well known to be completely negligible at the propagation frequency considered here, even when the density is as high as \( 10^6 \) electrons/cm³. The interplanetary medium between the earth and Venus is believed to have a density of less than \( 10^6 \) electrons/cm³ and, consequently, has little effect on the propagation-time measurements. The electron density and the extent of the Venussian ionosphere are at present entirely unknown, but most estimates give an upper bound of electron density within one or two orders of magnitude of that of the earth. It is of interest to compute the required mean electron density (equivalent to the mean group index of refraction) over the path from the radar to Venus that would cause an error in the determination of the a.u. of 100 km. This quantity is directly related to the number of electrons in a square-centimeter column containing the ray. Utilizing the simple theory outlined above, the resulting mean density is found to be \( 1.3 \times 10^7 \) electrons/cm³. Thus, if the ionosphere of Venus is assumed to be \( 10^7 \) km thick, the density would have to be approximately \( 2 \times 10^9 \) electrons/cm³, which is extremely high. Obviously, such an electron density yields a critical frequency above the propagation frequency, and the wave would have been reflected from the electron layer. Under such a circumstance, the theory employed here would have to be modified. However, several powerful arguments can be made to prove that the reflection was from the surface of the planet. A close inspection of all the Doppler residuals (see Fig. 1), for example, indicates that no short-term
changes in the Doppler velocity as large as 1 m/sec were observed in the course of the experiment; i.e., the observed Doppler frequency was always systematically different from the predicted frequency in a way completely explainable by the value of the a.u. used in the predictions except for system noise. Such a stability in the Doppler velocity seems completely at variance with what one would expect from a reflection from a soft ionic layer. Further, such a reflection undoubtedly would have an associated high loss in signal power, which is also contrary to observation. Thus, a dispersion effect as great as 100 km seems unlikely.

A more convincing argument against any important dispersion effects can be made by comparing the results of this experiment with those of a similar experiment by the group at the Millstone Radar Observatory of the Lincoln Laboratory, MIT (1961). The resulting value of the a.u. that they report, presumably using the Newcomb ephemerides and the standard velocity of light, is 149 597 700±1500 km, which is in remarkable agreement with our results. The propagation frequency employed in their experiment was 440 Mc. If the mean electron density computed above for an assumed error in our a.u. of 100 km is used with the above theory, one finds that the value which the Millstone group should have received would have differed from our result by 7000 km.

The above arguments are sufficiently strong to allow the conclusion that the reflection was from the surface of Venus and that the dispersion effects on our result are small.

The history of efforts to determine the precise radius of Venus is long and of no great interest to this report. The latest determination due to de Vaucouleurs (1960) is 6100±4 km, which was adopted for this work. It is interesting to note that the method of a.u. determination using ranging observations reported here involves the radius of Venus directly, whereas the method employing the Doppler observations is independent of the radius; i.e., the Doppler observations are a function of the velocity of the planet as a mass point. Thus, in principle, this experiment offers a means for determining the radius of Venus. An error in the radius of 100 km would cause an error in the astronomical unit of 350 km at conjunction. Some variations in the radius were studied, and it was found that the best agreement between the Doppler and range determinations of the a.u. was obtained with a radius of 6100±50 km. Smaller increments than 50 km were not studied because of the difficulty in comparing the results.

It is interesting to note that a reflection from an ionospheric layer would also be noticed at this point, since the results from the Doppler data would be unaffected, whereas the range data would indicate a different Venusian radius from the one reported (de Vaucouleurs 1960). This technique for studying the radius of Venus cannot be fully exploited until the problems in the ephemerides are resolved.

The observed values of the Doppler velocity were obtained by the method described in Sec. II. The system was designed to give optimum performance at the receiver threshold for a sine-wave input of slowly varying frequencies; i.e., the system gives an unbiased estimate of the input frequency changing with time in the manner predicted by ephemerides calculations. In practice, the signal presented to the Doppler-detecting mechanism was narrow-bandwidth noise of a few cycles in width centered about the true Doppler frequency. The response of this type of input is difficult to analyze, but the assumption of a linear system enables one to predict that the system will measure the center frequency of the input spectrum in an unbiased manner for frequency variations expected from the Venus echo. It should be realized, however, that a very small biasing in this measurement would be serious. Further study and experimentation in this area are desirable. As was pointed out in Sec. II, the two-way coherent nature of the system completely eliminated the effects of frequency drifts of the oscillators internal to the system. The range measurements were obtained in the manner described in Sec. II. The primary source of bias error to be expected in any ranging system is the uncompensated delays in the system. In this experiment, the delays were carefully calibrated by measuring the round-trip time between the transmitting antenna and the receiving antenna over a carefully surveyed microwave link. It is felt that this calibration procedure resulted in fixed delay errors of about 10 km (Easterling 1961). The range-data residuals presented in Fig. 2 indicate that a long-period sinusoidal error was present with the random fluctuations. This long-period component was due to the transient response of the extremely narrow-band system. The period of these oscillations was relatively short with respect to the time of observation (and computation) and, consequently, did not bias the results. The effect of biasing in this manner is estimated to be less than 100 km. Extensive study of this problem will continue.

Because of the somewhat speculative nature of this error analysis, it is very difficult to arrive at a probable error for the over-all astronomical unit determination. Therefore, it is felt that only an outer bound on the error of ±500 km should be specified.

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